The problem is to draw a cubic polynomial with a Bézier curve. The Bézier is defined in terms of u and the control points $\{x_0, y_0\}, \{x_1, y_1\}, \{x_2, y_2\}, \{x_3, y_3\}$.

$$x = a_3\ u^3 + a_2\ u^2 + a_1\ u + a_0$$

$$y = b_3\ u^3 + b_2\ u^2 + b_1\ u + b_0$$

$$x_0 = a_3\,(-1)^3 + a_2\,(-1)^2 + a_1\,(-1) + a_0$$

$$x_3 = a_3\ 1^3 + a_2\ 1^2 + a_1\ 1 + a_0$$

$$x_3 - x_0 = 2\ a_1 + 2\ a_3$$

Solve for a_1.

$$a_1 = -\frac{2\,a_3 + x_0 - x_3}{2}$$

$$x_0 + x_3 = 2\ a_0 + 2\ a_2$$

Solve for a_0.

$$a_0 = -\frac{2\,a_2 - x_0 - x_3}{2}$$

$$\frac{dx}{du} = a_1 + 2\ a_2\ u + 3\ a_3\ u^2$$

$$\frac{dx}{du} = -\frac{2\,a_3 + x_0 - x_3}{2} + 2\ a_2\ u + 3\ a_3\ u^2$$

$$\frac{dx}{du} = \frac{4\,a_2\,u + 2\,a_3\left(3\,u^2 - 1\right) - x_0 + x_3}{2}$$

$$\frac{x_1 - x_0}{\left(\frac{2}{3}\right)} = \frac{4\,a_2\,(-1) + 2\,a_3\left(3\,(-1)^2 - 1\right) - x_0 + x_3}{2}$$

$$\frac{3\,(x_1 - x_0)}{2} = -\frac{4\,a_2 - 4\,a_3 + x_0 - x_3}{2}$$

$$\frac{x_3 - x_2}{\left(\frac{2}{3}\right)} = \frac{4\,a_2\,1 + 2\,a_3\left(3 * 1^2 - 1\right) - x_0 + x_3}{2}$$

$$\frac{3\,(x_3 - x_2)}{2} = \frac{4\,a_2 + 4\,a_3 - x_0 + x_3}{2}$$

Solve for a_2.

$$a_2 = \frac{3\,(x_0 - x_1 - x_2 + x_3)}{8}$$

$$-\frac{3\,(x_0 - x_1 + x_2 - x_3)}{2} = 4\ a_3 - x_0 + x_3$$

Solve for a_3.

$$a_3 = -\frac{x_0 - 3\,x_1 + 3\,x_2 - x_3}{8}$$

To make the Bézier curve a cubic $a_3 = 0$ and $a_2 = 0$.

$$0 = -\frac{x_0 - 3\,x_1 + 3\,x_2 - x_3}{8}$$

$$0 = \frac{3\,(x_0 - x_1 - x_2 + x_3)}{8}$$

$$0 = \frac{2\,x_0 - 3\,x_1 + x_3}{4}$$

Solve for x_1.

$$x_1 = \frac{2\,x_0 + x_3}{3}$$

$$0 = \frac{x_0 - 3\,x_2 + 2\,x_3}{4}$$

Solve for x_2.

$$x_2 = \frac{x_0 + 2\,x_3}{3}$$

$$x = \frac{u\,(x_3 - x_0)}{2} + \frac{x_0 + x_3}{2}$$

$$u = \frac{2\,x - x_0 - x_3}{(x_3 - x_0)}$$

Solve for u_4 and u_5.

$$u_4 = \frac{2\,x_4 - x_0 - x_3}{(x_3 - x_0)}$$

$$u_5 = \frac{2\,x_5 - x_0 - x_3}{(x_3 - x_0)}$$

Define $u_0 = -1$ and $u_3 = 1$.

$$y_0 = b_3\ u_0^3 + b_2\ u_0^2 + b_1\ u_0 + b_0 \tag{1}$$

$$y_4 = b_3\ u_4^3 + b_2\ u_4^2 + b_1\ u_4 + b_0 \tag{2}$$

Subtract equation (1) from equation (2).

$$y_4 - y_0 = b_1\,(u_4 - u_0) + b_2\,\left(u_4^2 - u_0^2\right) + b_3\,\left(u_4^3 - u_0^3\right)$$

$$\frac{y_0 - y_4}{(u_0 - u_4)} = b_1 + b_2\,(u_0 + u_4) + b_3\,\left(u_0^2 + u_0\ u_4 + u_4^2\right)$$

Define y_{04}.

$$y_{04} = b_1 + b_2\,(u_0 + u_4) + b_3\,\left(u_0^2 + u_0\ u_4 + u_4^2\right) \tag{3}$$

Define y_{45}.

$$y_{45} = b_1 + b_2\,(u_4 + u_5) + b_3\,\left(u_4^2 + u_4\ u_5 + u_5^2\right) \tag{4}$$

Define y_{53}.

$$y_{53} = b_1 + b_2\,(u_5 + u_3) + b_3\,\left(u_5^2 + u_5\ u_3 + u_3^2\right) \tag{5}$$

Subtract equation (3) from equation (4).

$$y_{45} - y_{04} = b_2\,(u_5 - u_0) - b_3\,\left(u_0^2 + u_0\ u_4 - u_5\,(u_4 + u_5)\right)$$

Subtract equation (4) from equation (5).

$$\frac{y_{04} - y_{45}}{(u_0 - u_5)} = b_2 + b_3\,(u_0 + u_4 + u_5)$$

Define y_{05}.

$$y_{05} = b_2 + b_3\,(u_0 + u_4 + u_5) \tag{6}$$

Define y_{43}.

$$y_{43} = b_2 + b_3\,(u_4 + u_5 + u_3) \tag{7}$$

Subtract equation (6) from equation (7).

$$y_{43} - y_{05} = b_3\,(u_3 - u_0)$$

Solve for b_3.

$$\frac{y_{05} - y_{43}}{(u_0 - u_3)} = b_3$$

$$\frac{y_{43} - y_{05}}{2} = b_3$$

Solve for b_2.

$$b_2 = y_{05} - b_3 \, (u_4 + u_5 - 1)$$

Solve for b_1.

$$b_1 = -\frac{2\, b_3 + y_0 - y_3}{2}$$

Solve for b_0.

$$b_0 = -\frac{2\, b_2 - y_0 - y_3}{2}$$

$$b_2 = \frac{3\, (y_0 - y_1 - y_2 + y_3)}{8} \tag{8}$$

$$b_3 = -\frac{y_0 - 3\, y_1 + 3\, y_2 - y_3}{8} \tag{9}$$

Subtract equation (8) from equation (9).

$$b_3 - b_2 = -\frac{2\, y_0 - 3\, y_1 + y_3}{4}$$

Solve for y_1.

$$y_1 = -\frac{4\, b_2 - 4\, b_3 - 2\, y_0 - y_3}{3}$$

Add equation (8) to equation (9).

$$b_2 + b_3 = \frac{y_0 - 3\, y_2 + 2\, y_3}{4}$$

Solve for y_2.

$$y_2 = -\frac{4\, b_2 + 4\, b_3 - y_0 - 2\, y_3}{3}$$

Below is the cubic polynomial passing thru the points $\{x_0, y_0\}, \{x_4, y_4\}, \{x_5, y_5\}, \{x_3, y_3\}$.